

AD 740647

AD

REPORT NO. RG-TN-71-5

**DIGITAL REDESIGN OF A ROCKET ATTITUDE
CONTROL SYSTEM: PART II**

by
Sherman M. Seitzer

November 1971

Approved for public release, distribution unlimited.

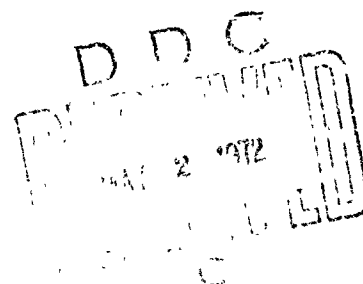


U.S. ARMY MISSILE COMMAND

Redstone Arsenal, Alabama

UC PRIOR REPORT
ON #1

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE



DISPOSITION INSTRUCTIONS

Destroy this report when it is no longer needed. Do not return it to the originator.

DISCLAIMER

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

ACCESSION BY	
CSFTI	WHITE SECTION <input checked="" type="checkbox"/>
DDC	BUFF SECTION <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
DIST.	AVAIL. AND/OR SPECIAL
A	

TRADE NAMES

Use of trade names or manufacturers in this report does not constitute an official indorsement or approval of the use of such commercial hardware or software.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Guidance & Control Directorate Dir for Res, Dev, Engr & Msl Sys Lab US Army Missile Command Redstone Arsenal, Alabama 35809		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP N/A	
3. REPORT TITLE DIGITAL REDESIGN OF A ROCKET ATTITUDE CONTROL SYSTEM: PART II			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Note			
5. AUTHOR(S) (First name, middle initial, last name) Sherman M. Seltzer			
6. REPORT DATE 30 November 1971		7a. TOTAL NO. OF PAGES 12	7b. NO. OF REFS 2
8a. CONTRACT OR GRANT NO. a. PROJECT NO. (DA) 1X222254D678 c. AMC Management Structure Code No. 4220.28.5465.1.31 d.		8b. ORIGINATOR'S REPORT NUMBER(S) RG-TN-71-5 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AD_____	
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Same as No. 1	
13. ABSTRACT A technique which enables a designer to perform digital redesign of an existing continuous control system has been expanded by Professor Benjamin C. Kuo of the University of Illinois to enable digital redesign of a forced control system. This report presents a brief description of the newly expanded technique and includes an example of its application.			

DD FORM 1473

REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED

Security Classification

11

Security Classification

12

Security Classification

30 November 1971

Report No. RG-TN-71-5

DIGITAL REDESIGN OF A ROCKET ATTITUDE CONTROL SYSTEM: PART II

by
Sherman M. Seltzer

DA Project No. 1X222254D678
AMC Management Structure Code No. 4220.28.5465.1.31

Approved for public release; distribution unlimited.

Guidance and Control Directorate
Directorate for Research, Development, Engineering
and Missile Systems Laboratory
U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

ABSTRACT

A technique which enables a designer to perform digital redesign of an existing continuous control system has been expanded by Professor Benjamin C. Kuo of the University of Illinois to enable digital redesign of a forced control system. This report presents a brief description of the newly expanded technique and includes an example of its application.

CONTENTS

	Page
1. Introduction	1
2. Theoretical Basis	1
3. Example	3
4. Conclusions	6

ILLUSTRATIONS

Figure		Page
1	Continuous Model	7
2	Sampled-Data Model	7
3.	Simplified Rocket Continuous Control System	7
4.	System Response to Step Input of Magnitude X	8
5	Simplified Rocket Sampled-Data Control System	8

1. Introduction

A technique under development by Professor Benjamin C. Kuo of the University of Illinois has been described and applied to several design examples.^{1,2} The technique enables the designer to perform a digital redesign of an existing continuous control system. The technique was originally developed for application to certain linear regulator systems. Recently Professor Kuo expanded the capability of the technique to enable digital redesign of a forced control system which enables the technique to be more appropriately applied to the models. This report describes briefly the extension of the technique introduced and includes an example of its application.

2. Theoretical Basis

Assume a linear continuous control system is described by the state variable equations,

$$\dot{\underline{x}}(t) = A\underline{x}(t) + B\underline{m}(t) \quad (1)$$

and

$$\underline{m}(t) = E(0) \underline{r}(t) - G(0) \underline{x}(t) \quad (2)$$

where $\underline{x}(t)$ is a vector representing the state of the system; A , B , $G(0)$, and $E(0)$ represent time-invariant matrices; and $\underline{r}(t)$ is a vector representing the input to the system.

The system may be represented by the block diagram shown in Figure 1. The objective of this investigation is to find a sampled-data system such that its response to an arbitrary input will be identical to the response of the continuous control system at the sampling instants.

Suppose it is desired to replace the continuous control system [represented by Figure 1 and Equations (1) and (2)] with a digital version. This version is achieved by adding a sampling device (with sample period T) followed by a zero order hold (also called a clamp or boxcar device). Such a control system is represented schematically in the block diagram of Figure 2 where $\underline{m}(kT)$ represents the output of the zero order hold (shown as the block labeled ZOH) and k is an integer.

¹Presented by Professor Kuo at a seminar in the Astrionics Laboratory, Marshall Space Flight Center, NASA, Huntsville, Alabama, 20-24 July 1970 (modified in subsequent correspondence between Professor Kuo and Dr. Seltzer).

²Seltzer, S. M., Digital Redesign of a Rocket Attitude Control System, U.S. Army Missile Command, Redstone Arsenal, Alabama, January 1971, Report No. RG-TN-71-1.

The goal of this approach is to make the sampled-data system of Figure 2 equivalent to the continuous system of Figure 1 in the sense that $\underline{m}(t)$ will be equal to $\underline{m}(kT)$, where t lies between kT and $(k+1)T$, and the state trajectories of the two systems, represented by the state Equations (1) and (3), are similar (in the sense implied above). Then the discrete data system may be represented analytically by

$$\underline{\tilde{x}}(t) = A\underline{\tilde{x}}(t) + B\underline{m}(t) \quad (3)$$

and

$$\underline{m}(kT) = E(T) \underline{x}(t) - G(T) \underline{\tilde{x}}(kT) \quad (4)$$

The state vector $\underline{\tilde{x}}(t)$ is used rather than $\underline{x}(t)$ to denote that the formerly continuous system now is sampled and has a different feedback matrix, $G(T)$, and input matrix, $E(T)$.

The development presented under Section 2 of Reference 2 remains initially unchanged, i.e., the following relationships still hold for $G(T)$:

$$G(T) = G(0) + T \left. \frac{\partial G(T)}{\partial T} \right|_{T=0}, \quad (5)$$

where

$$\left. \frac{\partial G(T)}{\partial T} \right|_{T=0} = \frac{1}{2} G(0) [A - B G(0)] \quad (6)$$

An expression for $E(T)$ is found in a similar manner, leading to the equation,

$$E(T) = E(0) + T \left. \frac{\partial E(T)}{\partial T} \right|_{T=0}, \quad (7)$$

where

$$\left. \frac{\partial E(T)}{\partial T} \right|_{T=0} = -\frac{1}{2} G(0) B E(0) \quad (8)$$

If $G(0)$, $E(0)$, A , and B are specified for the continuous model, $G(T)$ may be obtained for the sampled-data model by applying Equations (5) through (8).

3. Example

A primitive linearized model of a rocket dynamical continuous control system may be characterized by the state Equations (1) and (2) where

$$\underline{x} = [\theta, \dot{\theta}]^T, \quad (9)$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad (10)$$

$$B = [0, c]^T, \quad (11)$$

$$E(0) = 1, \quad (12)$$

and

$$\begin{aligned} \underline{r}(t) &= \chi, \quad t \geq 0 \\ &= 0, \quad t < 0 \end{aligned} \quad (13)$$

The corresponding block diagram representing this simple time-invariant system is shown in Figure 3. For a particular analytical model of a rocket system being studied by the U.S. Army Missile Command, a value of c corresponding to passage of the vehicle through maximum dynamic pressure conditions would be 90 [in units consistent with the other terms used in Equation (1)]. To obtain values for control gains a_0 and a_1 that will yield desirable response characteristics, the characteristic equation corresponding to this system may be obtained from the relation,

$$\left| sI - \bar{A} \right| = 0, \quad (14)$$

where

$$\dot{\underline{x}} = \bar{A} \underline{x} + B \chi \quad (15)$$

and

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ -a_0 c & -a_1 c \end{bmatrix} . \quad (16)$$

Equation (14) leads to the characteristic equation,

$$s^2 + a_1 c s + a_0 c = 0 , \quad (17)$$

where the natural frequency ω_n and damping ratio ζ may be written as

$$\omega_n = \sqrt{a_0 c} \quad (18)$$

and

$$\zeta = \frac{1}{2} a_1 \sqrt{\frac{c}{a_0}} . \quad (19)$$

If a value of 6 radians/second is chosen for ω_n and $1/\sqrt{2}$ for ζ ,

$$G(0) = [a_0, a_1] = [0.398, 0.094] . \quad (20)$$

Using these values for $G(0)$, the expected response to a step input λ may be written as

$$\begin{aligned} \frac{\theta(t)}{\lambda} &= \mathcal{L}^{-1} \left\{ \frac{c}{s(s^2 + a_1 c s + a_0 c)} \right\} \\ &= 3.54 e^{-4.25 t} \cos(\omega t + 135 \text{ deg}) + 2.5 \end{aligned} \quad (21)$$

and is plotted in Figure 4. Application of the final value theorem leads to the steady-state value of λ/a_0 for $\theta(t)$.

Applying the technique described in Paragraph 2, the continuous system of Figure 3 may be converted to a sampled-data system as indicated in Figure 5. Equations (3) and (4) may be used to describe the system and Equations (6), (8), (10), (11), (12), and (20) used to obtain the expressions,

$$\left. \frac{\partial G(T)}{\partial T} \right|_{T=0} = \frac{1}{2} G(0) [A - BG(0)] = \frac{c}{2} \left[\begin{pmatrix} -a_0 & a_1 \end{pmatrix}, \left(\frac{a_0}{c} - a_1^2 \right) \right]. \quad (22)$$

and

$$\left. \frac{\partial E(T)}{\partial T} \right|_{T=0} = -\frac{1}{2} G(0) BE(0) = -\frac{1}{2} a_1 c. \quad (23)$$

Using Equations (5) and (22), an expression for the feedback gains may be written:

$$G(T) = \begin{bmatrix} \tilde{a}_0, \tilde{a}_1 \end{bmatrix} = \begin{bmatrix} a_0 \left(1 - \frac{a_1 c T}{2} \right), a_1 + \frac{1}{2} c T \left(\frac{a_0}{c} - a_1^2 \right) \end{bmatrix}. \quad (24)$$

Similarly, using Equations (7) and (23), an expression for $E(T)$ may be obtained:

$$E(T) = 1 - \frac{1}{2} a_1 c T. \quad (25)$$

Using Figure 5 and Equations (24) and (25), the system transfer function may be found:

$$\begin{aligned} \frac{\theta(z)}{X(z)} &= \frac{z \left\{ \left(\frac{1 - e^{-sT}}{s} \right) \left(\frac{c}{s^2} \right) \right\} E(T)}{1 + z \left\{ \left(\frac{1 - e^{-sT}}{s} \right) \left(\frac{c}{s^2} \right) (\tilde{a}_0 + \tilde{a}_1 s) \right\}} \\ &= \frac{c T^2 E(T) (z + 1)}{2z^2 + \left(\tilde{a}_0 c T^2 + 2\tilde{a}_1 c T - 4 \right) z + \left(\tilde{a}_0 c T^2 - 2\tilde{a}_1 c T + 2 \right)}. \end{aligned} \quad (26)$$

Application of the final value theorem to Equation (26) leads to the steady-state value of X/a_0 for θ when the system input is a step of magnitude X .

If the numerical values of Equation (20) are used for a_0 and a_1 and c is set at 90, $G(T)$ and $E(T)$ are found to be:

$$G(T) = [(0.398 - 1.692 T), (0.094 - 0.2 T)] = [\tilde{a}_0, \tilde{a}_1] \quad , \quad (27)$$

and

$$E(T) = 1 - 4.2488 T \quad . \quad (28)$$

To preclude positive feedback, T must have a numerical value less than $0.398/1.692 = 0.24$.

Using the results of Equations (27) and (28), Equation (26) may be rewritten as

$$\theta(nT) = z^{-1} \{ \theta(x) \} = z^{-1} \left[\frac{90T^2(1 - 4.2488T)(s + 1)X(s)}{2s^2 + [(39.8 - 152.3T)T^2 + (16.9 - 36.0T)T - 4]s + [(39.8 - 152.3T)T^2 - (16.9 - 36.0T)T + 2]} \right] \quad (29)$$

For a step input of magnitude X ,

$$X(z) = \frac{Xz}{(z - 1)} \quad . \quad (30)$$

Values of $\theta(nT)$ may be found from Equations (29) and (30) for various selected sampling periods, T (Figure 4).

4. Conclusions

The technique for performing digital redesign of a missile attitude control system that can be formulated in the form of Equations (1) and (2) is simple and easy to apply. As the chosen value for T

approaches the limiting value of 0.24, the step input response (θ) becomes more sluggish, becoming unstable when $T > 0.24$ (Figure 4).

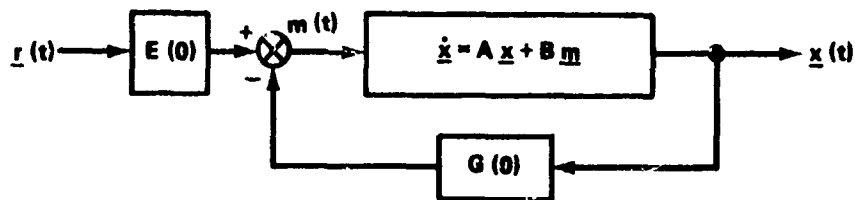


Figure 1. Continuous Model

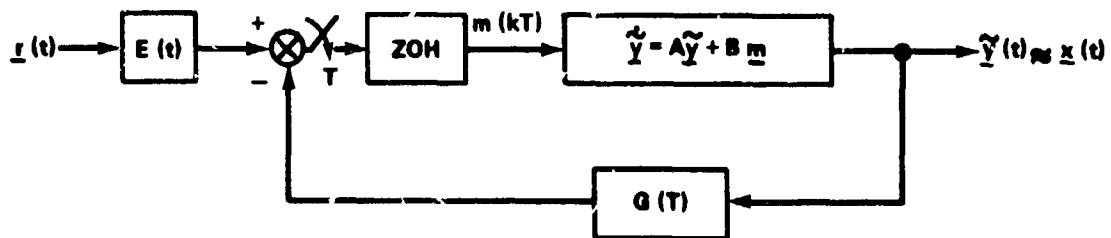


Figure 2. Sampled-Data Model

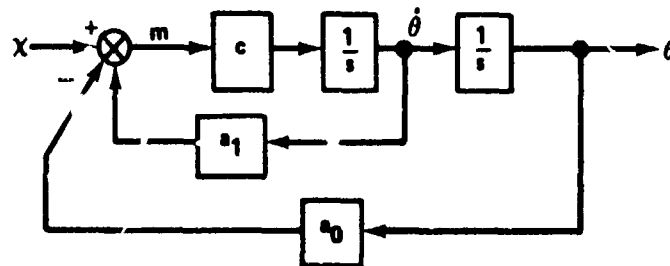


Figure 3. Simplified Rocket Continuous Control System

DISTRIBUTION

	No. of Copies
U.S. Army Missile Command Distribution List A	52
National Aeronautics and Space Administration ATTN: S&E-ASTR-DIR, F. B. Moore	1
-A, S. M. Seltzer	5
H. Hosenthien	1
Marshall Space Flight Center Alabama 35812	
University of Illinois Department of Electrical Engineering ATTN: Professor Benjamin Kuo Champaign, Illinois 61820	1
AMSMI-O	1
-R, Dr. McDaniel	1
-RB, Dr. Kobler	1
-RC	1
-RD	1
-RE	1
-RF	1
-RG, Mr. Huff	1
-RK	1
-RL	1
-RN	1
-RR	1
-RT	1
-RCN	5
-KBL	5
-RPR	1
-RG (Record Set)	1
-RP, Mr. Kopcha	1
Defense Documentation Center Cameron Station Alexandria, Virginia 22314	12